Master Thesis for Master of Philosophy in Environmental and Development Economics

Fossil Fuel Extraction Under Climate Policy

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Preface

I am deeply grateful to both of my supervisors, Professor Michael Hoel and Professor Olav Bjerkholt. Words alone are not enough to express how much I want to thank them for their guidance for my thesis writing.

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Abstract

This paper reviews some main studies on fossil fuel extraction under climate issues and studies a theoretical model of monopoly extraction under pollution stock ceiling constraint. We show that under constant elasticity demand and zero extraction cost, the monopolist will behave exactly the same as in the competitive case, and the existence of the ceiling constraint will initially push the extraction to grow at a rate higher than the interest rate in both monopoly and competitive case; With a non-zero extraction cost the monopoly may be under lower risk to be affected by the ceiling than the competitive case; With a constant non-zero cost, which can be either extraction cost or abatement cost, the comparison monopoly versus competitive market is identical to the comparison "social optimum with high cost" versus "social optimum with low cost".

Introduction

The environmental problems caused by combustion of fossil fuels are directly connected to what we've already heard a lot: global warming. Global warming plays a major role in causing severe weather events around the world, which happens continually recent years and causes both human and economic loss. The threats posed by global warming, such as hurricanes, typhoons, tornadoes, blizzards, droughts, floods and other catastrophes, are becoming harder and harder to ignore. Polluting the atmosphere could be a key factor in increasing temperatures around the world. Greenhouse gases, of which carbon dioxide are of high abundance, generated largely by combustion of fossil fuels, trap sun's heat in atmosphere and have been linked to rising global temperature. Renewable energy is not replacing fossil fuels as quickly as scientists have been forecasting, people are consuming more and more oil, coals…every year. Behind the high speed economic development is the danger of overpolluting, which when accumulated to certain amount will cause irreversible environmental tragedies.

"The Stern review on the economics of climate change" is a 700-page report released on October 2006 by economist Lord Stern, which discussed the effect of climate change and global warming on the world economy. Estimating the overall effects from the cost side, the review calculated that the dangers of unabated climate change would be equivalent to 5% of the world GDP each year, yet the costs of reducing greenhouse gas emissions to avoid the worst impacts of climate change could be limited to around only 1% of global GDP each year. And if we don't take action, "the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP each year, now and forever". In other words, reducing emissions to tackle with climate changing would make us better off. As the most widely studied and discussed report in climate change and economic development, the Stern report significantly drew the world's attention to the issues of climate change. Focusing on the cost side, the review couldn't make it clearer that climate change could have serious impacts on human development, and if we take action now, there is still time to avoid the worst impacts to happen.

The European Economic Advisory Group Report (2008) on the European economy pointed out that to find useful policies to control carbon emissions, not only the cost side, but also the supply side must be considered, because supply reactions in carbon markets were fundamentally different from that in normal markets for the simple reason that the stock of carbon in the ground is exhaustible and can't be reproduced. To date the public climate policy discussion has been focused on the reduction of demand for fossil fuels. However, demand reducing measures such as emission permits and the subsidization of alternative energy sources risk being ineffective or even counterproductive without proper analysis from supply side.

Classical studies in exhaustible resource extraction, started from Hotelling (1931), have been mainly focused on the optimal allocation of the resource under finite supply over time horizon. Based on the classical Hotelling model, a two state variable dynamic with pollution externality has been developed since the 1990s and the dynamic has been maintained in recent studies which combine climate policy into the control process. Most recent studies on fossil fuel extraction under climate policies assume perfectly competitive market, while the monopoly case hasn't yet been specifically studied. However, the geographic nature of the stock of fossil fuels such as oil makes its supply market more likely to be monopolistic than perfectly competitive, as is the truth for the world oil market today.

This paper studies a simple case of monopoly extraction under a pollution stock ceiling and compares that with a competitive market. The results show that under constant elasticity demand and zero extraction cost, the monopolist will behave exactly the same as in the competitive case, and the existence of the ceiling constraint will initially push the extraction to grow at a rate higher than the interest rate in both monopoly and competitive case; With a non-zero extraction cost the monopoly may be under lower risk to be affected by the ceiling than the competitive case; With a constant non-zero cost, the comparison monopoly versus competitive market is identical to the comparison “social optimum with high cost” versus “social optimum with low cost”.

The remainder of this paper develops as follows: Chapter 1 briefly reviews the classical Hotelling theory on nonrenewable resource extraction. Chapter 2 reviews some earlier studies on fossil fuel extraction that included the pollution externality into the model. Chapter 3 reviews some recent studies on the fossil fuel extraction under climate policy. Chapter 4 initiates a study on the monopoly extraction under pollution stock ceiling with constant elasticity demand and zero extraction cost, and briefly extends the discussion to the case when there exists constant non-zero cost.

Chapter 1 Classical Hotelling theory on non-renewable resource extraction

The optimal extraction of non-renewable resources, mainly fossil fuels, is one of the main branches in resource economics. Optimal control theory is extensively applied in the area to solve for the optimal path of the resource extraction over a time horizon. Hotelling (1931) characterizes the beginning of the era for the study of exhaustible resource extraction. In a typical Hotelling model of optimal extraction, the resource rent, or the marginal profit of extracting, grows at a rate that equals the discount rate. Define P_t as the market price received and R_t as the amount extracted at time t. The time variable is dropped where convenient. The inverse demand function is given by $P = P\left(R\right)$. In a perfectly competitive market, the social utility function is defined by $U(R) = \int_0^R P(\tau)d\tau$. With zero extraction cost, the social planner aims to maximize the total utility of all periods: $\max \int_0^\infty U(R)e^{-rt}dt$.[1] The current stock of the resource is defined by S_t, which is the state variable of the control problem. The rate of change of the resource stock is given by $\dot{S}_t = -R_t$, where R functions as the control variable. The current value Hamiltonian $H = U(R) - \lambda R$. From the first order condition $U^{'}(R) = \lambda$ and the time derivative for the costate variable $\dot{\lambda} = r\lambda$, one can get $\frac{\dot{P}}{P} = r$. If given an initial price P_0, $P_t = P_0 e^{rt}$ fixes the relative prices for the resource at different times. The total stock of the non-renewable resource is given by S_0. Suppose the resource is exhausted at time T.[2]

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[1]If the resource is exhausted at a finite time T, then $U(R) = 0$ for $t \geq T$.

[2]For the resource to be optimally exhausted at a finite time, the price at zero demand must not be infinitely high. The resource is exhausted in infinite time if $P(0) = \infty$.

P_0 and T can be co-determined by $\int_0^T R dt = \int_0^T f\left(P_o e^{rt}\right) dt = S_0$ and $R_T = f\left(P_0 e^{rT}\right) = 0$, where $f(P) = P^{-1}(P)$.

Following the same procedure, for a monopolist who is supposed to maximize the total profit $Y = PR$ of all times, the optimal path will be such that the marginal profit grows at a rate that equals the interest rate r, that is, $\dot{Y(R)'} = rY(R)'$; Under non-zero extraction cost for both social planner and the monopolist, the optimal path will be such that the net marginal profit grows at a rate of r.

The Hotelling's model describes a single state variable optimal control problem, which aims to obtain the optimal path that maximizes the objective function under the exhaustion constraint. The nature of difference in optimal paths for competitive market and monopoly is that the objective functions differ in the two cases. For competitive market the objective function is the social total utility, while in monopoly case it is the total profit. Since Hotelling's study, the optimal resource extraction problems have been studied dynamically, as it should be due to the scarcity nature of the non-renewable resources. The optimal control theory is the mathematical basis for studies in the area.

Chapter 2 Fossil fuel extraction with pollution externality

Dynamic exhaustible resource extraction study is incomplete without taking into consideration of continues development of society and human being. “The economics of climate change and the economics of exhaustible resources could not be more closely intertwined, for in essence the problem of global warming is the problem of gradually transporting the available stock of carbon from underground into the atmosphere, with useful oxidization on the way.”(Sinn (2007))

For the last decades, the study of exhaustible resource extraction has been extended from the classical Hotelling exhaustible resource model to models with environmental concerned aspects. In most of those studies, environmental damages caused by the consumption of exhaustible resources, mainly fossil fuels, were modeled as a loss of the social welfare. This negative externality that causes pollution to the environment from burning fossil fuels was included in the objective function in form of a damage function. Define Z as the stock of pollution in the atmosphere. Since carbon dioxide is the main greenhouse gas, Z is also referred to as carbon stock. Assume competitive market, a general model in the fossil fuel extraction with pollution externality can be described as follows:

$$
\begin{aligned}
&\max \int_0^{\infty} \left(U(R) - C(R) - D(Z)\right) e^{-rt} dt \\
&s.t. \dot{S} = -R \\
&\dot{Z} = aR - bZ \\
&R \geq 0
\end{aligned}
$$

The social utility function $U(R)$ is assumed to be concave. $C\left(R\right)$ is the cost function. $D\left(Z\right)$ is the damage function of pollution stock, which is assumed to be increasing and convex. a is the polluting emission proportion from consuming R unit of fossil fuels, and b is the natural regeneration parameter of the atmosphere. The time horizon is set to be infinite. Under a concrete demand function, if the resource is exhausted at a finite time T, then $R = 0$ for $t > T$.

One of the main challenges of solving this control problem might be technical. Problem arises as a result of the fact that the extraction amount also controls the path for the growth of the stock of pollutants. Withagen (1994) made it clear that an optimal control problem in the presence of stock externality as modeled in the above is a two state variables control problem. He generally studied the extraction path for exhaustible resource in the presence of negative externalities by including a damage function of stock pollution, employing optimal control theory for two state variables. He employed the above model where $C(R) \equiv 0$. He compared the optimal extraction path when the pollution is causing damage with that when there is no pollution externality. The current value Hamiltonian would then become: $H = U(R) - D(Z) - \lambda R + \mu(aR - bZ)$. He compared the first order Hamiltonian with that where $D(Z) \equiv 0$. By doing so he showed that the initial extraction amount R_0 would be less than in the case of no negative externalities, and the two extraction paths intersect at a certain time point, the resource would be depleted at a lower rate than without pollution externality. Withagen (1994) is an early attempt in solving for the optimal exhaustible resource extraction in the presence of pollution externality. An important result from his work is that the initial extraction would be less than without externality in order to balance the pollution damage caused by the consumption of the resource.

Ulph and Ulph (1994) extended the study of the path for carbon tax with pollution externality and studied the effort of the pollution externality on the dynamic of carbon tax. They generalized the usual stock externalities analysis to allow the resource that generated the pollution be exhaustible. If we define the benefit function in Ulph and Ulph’s model as the total social welfare, their study is in fact equal to the study of exhaustible resource with pollution externality. The Hamiltonian of the control problem will then be:

$$H = U(R) - C(R) - D(Z) - \lambda R + \mu(aR - bZ)$$

where the costate variable μ is defined as the optimal carbon tax. Ulph and Ulph assumed a special case where benefit and damage functions were in quadratic form and showed that if the initial pollution stock was small, the carbon tax would be rising first and then falling until exhaustion. Carbon tax is a regulating tool for the control of carbon emissions by policy makers. They pointed out from the study that it was the time structure of the carbon tax rather than its level that would have an influence on the optimality.

Hoel and Kverndokk (1996) modeled the exhaustibility of the resource differently. They used a stock dependent marginal cost function, $C(S)$, and from the optimality they showed that the optimal carbon tax would be increasing at the beginning and then start to decrease before the carbon stock reached the maximum point. This result is consistent with that of Ulph and Ulph (1994). In addition, they related the behavior of the optimal carbon tax to the evolution of the carbon stock in the atmosphere and showed that the behavior of the carbon tax over time would depend on the initial growth of the carbon stock in the atmosphere. They pointed out that the carbon tax started to decline at a time before the carbon stock declined, and would continue to decline. That is, when μ begins to decline, it is only possible for Z to either also begin to decline or continue to increase before it declines.

The running of industries in modern society is largely reliable on the use of fossil fuels. Since it is not realistic to solely reduce energy use, scientists and policy makers are trying to find solutions for a possible transmission from highly polluted fossil fuels to less polluted or clean energy source. A clean backstop can provide the ideal energy substitute without polluting the atmosphere. However, due to limited techniques, the extraction cost of such a clean backstop might be much higher than fossil fuels. The substitution of fossil fuels and possible backstop technology is thus also a concern in the study of fossil fuel extraction with pollution externality.

Possibilities for substitution may affect the original path in certain ways. Dasgupta and Stiglitz (1981) had a discussion on how a future date when possible new technology appears would change the rate of depletion of the exhaustible resource. They studied the competitive market case, and showed that if the date of this new technology was uncertain, the rate of extraction ought to be chosen in such a manner that the resource stock keeped positive as long as the innovation had not occurred. Both cases when the amount of resource is large and small are considered. When the resource stock is large, the initial depletion rate is slower in most cases. This is an earlier study on the effect of possible backstops, assuming the substitute only appears in an uncertain future time instead of being available from beginning. The substitute, also referred to as backstop technology, when available in unlimited quantities can be clean energies such as solar and wind power, hydropower, nuclear power, which already exist. Instead of possibly being found in some future time, those known backstop energy are available initially when the extraction path is planned. In the existence of such non-polluting backstop technology, Hoel and Kverndokk (1996) made a comparison of the extraction path of fossil fuels with greenhouse externalities with that without externalities. The extraction paths will be of same pattern for with and without externalities, yet with externality the extraction will be slower. Also, in the existence of greenhouse externality, it would be optimal for consumer to consume both fossil fuels and the backstop when the price of fossil fuels reaches the price of the backstop. This result is different from classical backstop theories, where the resource is depleted until the price reaches the cost of the backstop, and the consumers will switch to backstop at that price immediately.

Part of Tahvonen (1997)' results also showed that instead of a clear cut out of fossil fuels and the backstop, there might be simultaneous use of both in certain conditions as a result of the stock externality. The substitution of fossil fuels and the backstop under pollution externality was extensively studied in Tahvonen (1997). He used the common way to model the backstop technology. Define $Q\ (\geq 0)$ as the backstop consumption, and m the constant marginal cost of the backstop. Including the consumption of the backstop into the objective function, the model can be written as follows:

$$\max \int_0^\infty \left(U(R+Q) - C(R) - mQ - D(Z)\right) e^{-rt}\, dt$$

$$s.t. \dot{S} = -R, R \geq 0$$

$$\dot{Z} = aR - bZ$$

Tahvonen first showed what the paths were like for carbon tax, pollution stock and resource extraction under a given initial pollution stock Z_0. When Z_0 is low enough, the carbon tax and the pollution stock have inverted U-shaped paths, while resource extraction decreases monotonically towards zero; With Z_0 high enough, the carbon tax and the pollution stock converge monotonically towards zero, while the resource extraction first increases and then decreases towards zero. These results showed the connection among the time paths for carbon tax, stock pollution and resource extraction. He showed the paths' dynamics was affected by the initial level of stock pollution, which was neglected in most works.

Later on he showed the possible combination of consuming fossil fuels and the backstop. Define Q^* as the optimal backstop use when $R = 0$, Q^* satisfies $U^{'}(Q^*) = m$. He showed there were three regimes for optimal use of the resource and the backstop: (1) $Q = 0, R > Q^*$; (2) $Q \geq 0, R \geq 0$ and (3) $Q = Q^*, R = 0$. The optimal resource consumption strategies could be a combination of the three regimes. As can be seen in regime (2), the consumption of both resources is positive, meaning the fossil fuels and the backstop energy can be simultaneously used. These combination uses of energies can't happen in the case where there are no pollution externalities.

A main challenge for studying pollution externality could be that it is empirically difficult to measure the exact damage function, and as a result the studies are done rather through general theoretical analysis than empirical modeling.

Chapter 3 Fossil fuel extraction under climate policy

The Kyoto Protocol Treaty, which was negotiated in 1997 at Kyoto, Japan and came into force in 2005, is an international agreement linked to the United Nations framework convention on climate change (UNFCCC). The Kyoto Protocol is generally seen as an important first step towards a truly global emission reduction regime. It sets binding targets for industrialized countries and the European community to reduce their collective greenhouse gases by an average of five percent against the year 1990 level over the period 2008-2012. (If compared to the emissions level that would be expected by 2010 without the Protocol, this target represents a 29% cut.)

On one hand, the issue of global warming is seeking more and more attentions. The impact of global warming on the world economy has been studied in a wide range, among which the Stern report is the most influencing one. The report pointed out that it was high time for human beings to take action against global warming. Although, unlike most studies that emphasize the urgency of global warming, Chakravorty, Roumasset and Kinping (1997) used empirical simulation in a multi resource and multi demand model and showed from the results that the issue of global warming might be overstated. They used empirical data to do a simulation on the use of energy substitutions under the concern of global warming. They generally extended the study of fossil fuel extraction and global warming into a situation with alternative regimes of technology change. In reality, except solar energy, the other clean energies are in fact also exhaustible. And instead of homogenous demand for a single resource, there may be simultaneous use of different fossil fuels for diverse demands in different industry areas. Their analysis also indicated that the transition to backstop technology might be the only viable solution to the threats of global warming.

On the other hand, recent studies of non-renewable resource extraction have been focused on the optimal path under the climate policy, which for example can be a climate policy such as the Kyoto Protocol. The characteristic of these studies is that the damage function is excluded from the model, and a upper limit on the pollution stock is added. This can also be seen as a special case of a convex damage function, where the damage function is zero until it goes to infinity at the time the stock pollution reaches the ceiling. The typical model studied is as follows:

$$\max \int_0^{\infty} \left(U(R) - C(R)\right) e^{-rt} dt$$
$$s.t. \dot{S} = -R$$
$$\dot{Z} = aR - bZ$$
$$R \geq 0$$
$$\overline{Z} \geq Z$$

Given the initial stock of pollution Z_0, the cumulative pollution is not allowed to exceed the ceiling $\overline{Z}$ that is set by the policy maker.

Assume there is only one resource and the extraction cost is zero. The Hamiltonian is

$$H = U(R) - \lambda R + \mu(aR - bZ) + q(\overline{Z} - Z)$$

Following Chakravorty, Moreaux and Tidball (2006)'s results, when $Z_0 = \overline{Z}$, the ceiling will be binding for a non-zero time from the beginning, and after the binding period the price path is growting as in the Hotelling path. When $Z_0 < \overline{Z}$, that is, the initial stock pollution is lower than the ceiling, the pollution stock rises from Z_0 to $\overline{Z}$, while the price path follows a non Hotelling path, and after the ceiling is binding for a non-zero time, the path is again pure Hotelling. The existence of the ceiling will result in the price path not develop as the traditional Hotelling path.

Chakravorty, Moreaux and Tidball (2006) also extended the situation to that with the existence of imperfect polluting resource substitutes. They assumed there simultaneously existed one high cost (for example natural gas) and one low cost (for example coal) nonrenewable resource. They showed that it might be efficient to extract the more polluting yet lower cost resource like coal first, then natural gas and finally again coal. The pattern of the extraction would be dependent upon the initial endowment of the two resources. If the stock of coal is relatively low, then the Hotelling rents of the two resources are exactly equal and regulation is never binding. If coal is abundant, it has a lower Hotelling rent than natural gas. With abundant resources, “extraction paths have a turnpike feature in which both resources are jointly extracted at the maximum allowed level.” Chakravorty, Moreaux and Tidball (2006) can be considered as a first step towards the understanding of the affects of the environmental regulation on the extraction of non-renewable resources.

The substitution of fossil fuels and non-polluting backstop energy under stock pollution ceiling was extended in Chakravorty, Magné and Moreaux (2006) to the case of non-stationary demand and abatement activity. They assumed two resource, polluting coal and clean solar energy. In addition they assumed that carbon emissions could be abated at constant unit cost, such as through sequestration by forests or pollution reduction at source. Assuming an exogenous stock pollution ceiling, they studied the price paths under stationary, increasing and decreasing demand for coal and solar energy in perfectly competitive market. Their results showed that abatement took place only at the ceiling. In all cases, coal is used exclusively in the initial period. The stock of pollution builds up over time, followed by an interval in which the ceiling is binding. After that the stock pollution declines to zero gradually.

Besides the backstop technology, public policies may also be efficient tools to mitigate the problem of global warming. Such policies “must succeed in flattening the carbon supply in the world energy market” (Sinn (2007)). Sinn pointed out that useful policies would include public finance measures to flatten the supply path, safer property rights, binding quantity constraints and technical means to decouple the accumulation of carbon dioxide from carbon consumption. Sequestration is useful but difficult in reality due to the large quantities involved. This might indicate a very high abatement cost for the abatement activity suggested in Chakravorty, Magné and Moreaux (2006).

Chapter 4 A model of a resource monopoly under pollution stock ceiling

4.1 Monopoly and the climate policy

So far, most studies have assumed the fossil fuels being produced in competitive market. However, in the world oil market, monopoly may an extreme but alternative assumption.

The Organization of the Petroleum Counties (OPEC) plays as the biggest Cartel in the world oil market. OPEC has the considerably strong power in controlling the oil output and price in the international market. Recently years the world oil price keeps fluctuating, the United States is in a recession and much of the rest of the world faces an economic slowdown. If we see the world oil market as a big monopoly, then not only the world economy is greatly influenced, the pollutions caused by the production and consumption of oil is in high time to be regulated. The monopolist will then be limited from freely extracting the polluting resource under a regulation that set restriction on emission. In the case of oil market, government usually has the power to set the level of production and price, and therefore functions as a policy maker.

Take the oil market in China as an example. On July 2008, developing nations led by China and India rejected a proposal by G8 leaders to tackle climate change. The plan would see greenhouse gas emissions cut by 50% by 2050. As a matter of fact, oil is defined as a strategic resource in China. In spite of the release of Measures for the Administration of the Refined Oil Market and Measures for the Administration of the Crude Oil Market on January 1, 2007, foreign and private oil companies still face an oil market in China that is dominated by massive monopoly. Before, the refined oil market was controlled by China National Petroleum Corp. (CNPC) and China Petroleum & Chemical Corp. (Sinopec), and the crude oil was distributed collectively by the government. “As major suppliers of oil, CNPC and

Sinopec enjoy absolute control over the Chinese refined oil market due to the comprehensive operational and sales network they have established through years of monopoly."[3] The opening of the wholesaling market does not mean that the supervision by the government will be loosened. Oil companies cannot price oil freely. The price is set by the government who when decides the price needs to take consideration of many aspects, therefore the state-owned companies, which have absolute control over domestic oil sources, would be unlikely hit hard by the more competitive sort market.

One big concern in setting oil price for the government would be a responsibility in responding to climate change. The article by Gørild Heggelund (2007) demonstrates that prospects for emission reduction are not realistic under the current policy environment, and China is unlikely to take on commitments in the near future. However, China is now an active participant in the Clean Development Mechanism and is seeking to find a way to balance the emerging climate changing with economic development.

From truth of the oil market, monopoly is still a big topic in the world resource extraction and economics development. This is also true when it takes the form of local monopoly, as is the case in China. Although in reality the oil market rather take the form of oligopoly and monopolistic extraction, it is usually convenient to start from the extreme assumption of pure monopoly case. Most of the literature that study fossil fuel extraction under climate policy have assumed competitive market and almost none has specifically studied how a climate policy set to regulate the pollution may affect monopoly extraction.

[3] http://www.bjreview.com.cn

4.2 The basic model:

Consider the monopoly case with a single non-renewable resource. The initial time $t = 0$ is defined as the time when the policy is announced to the producer. Before this time, the extraction path would not be restricted by additional constraint and would follow the traditional Hotelling rule. Correspondingly, S_0 in the model studied will then be the amount of resource stock available at the time 0, therefore S_0 is actually the total stock of the fossil fuel minus the amount that has been extracted before the time the policy is announced. Z_0 is the stock of pollution when the policy is announced at time 0, and R_0 is the endogenous initial extraction amount at time $t = 0$.

Suppose the policy maker knows the "natural peak" of the stock of pollution, which is the maximal amount of carbon stock under the Hotelling path without a ceiling constraint. He will then set the ceiling somewhere below this natural maximal amount. [4] At time $t = 0$ the owner of the resource is aware of the ceiling constraint. If instead of taking action to adjust the extraction path, the producer still follows the previous Hotelling path, by the time the pollution stock reaches the ceiling, he will face a sudden cut down of extraction amount in compliance to the policy. In fact the extraction would then be cut down to such a level that the growth rate of the stock pollution Z_t equals zero. Such a kink in the extraction amount will lead to a sudden cut down of the objective utility or profit as well, and thus will not maximize the objective utility or profit function since optimization requires concavity and continuity of the objective function.

[4] A policy is effective only when the ceiling is set below the natural maximal pollution stock. Here the initial available fossil fuel is supposed to be abundant enough so that the emissions of carbon dioxide will potentially cause environment problems, which makes it necessary to set a climate policy.

Consider a demand curve with constant demand elasticity, the inverse demand function is denoted by $P = R^{\alpha-1}$, $(0 < \alpha < 1)$, where $\frac{1}{1-\alpha}$ is the elasticity of demand. In the case of monopoly, the elasticity of demand is assumed to be greater than 1. Since $0 < \alpha < 1 \Rightarrow \frac{1}{1-\alpha} > 1$, the assumption is satisfied.

At time $t = 0$ a total stock of the resource S_0 is available, and the stock of carbon dioxide in the atmosphere is known as Z_0. Let Z_t represent the stock of pollution in the atmosphere at period t, and $\overline{Z}$ be the ceiling on the pollution stock imposed exogenously by the regulator. An amount of R_t units of fossil fuel is extracted at period t. Let S_t be the amount of resource stock available at time t, so the equation of motions for the resource stock can be written as $\dot{S}_t = -R_t$. The stock of resource decreases over time, indicating the scarcity of the non-renewable resource.

Assume the carbon emission of the fossil fuel is proportional to its amount of consumption, this proportion is denoted by $a(>0)$. The atmosphere has a natural capability of recovering from pollution, let $b(>0)$ denote the nature regeneration capacity parameter of the atmosphere that is proportional to the stock of pollution. At time t, the growth rate of the pollution stock is given by $\dot{Z}_t = aR_t - bZ_t$. Furthermore, during the period when the ceiling is binding, $\dot{Z}_t = aR_t - bZ_t = 0$, thus $R \equiv \frac{b}{a}\overline{Z}$. The extraction amount keeps at the level of $\frac{b}{a}\overline{Z}$ until the binding ceases.

Consider the policy is set to control the pollution stock to “never exceed $\overline{Z}$ in future time”, indicating $\overline{Z} \geq Z_t$ for $t > 0$. In the optimal path under the ceiling constraint there will be a time θ_1 when the pollution stock reaches the ceiling, the ceiling is binding until time θ_2, later on the stock pollution will decline as time goes to infinity. θ_1 and θ_2 are endogenous. Define $Y(R)$ as the profit function for monopoly, $Y(R) = PR = R^{\alpha}$. Assume a constant marginal cost c. Interest rate is r. The monopoly problem is to maximize the total net profit given resource and pollution constraints. The control problem is as the following:

$$\max \int_0^{\infty} \left(R^{\alpha} - cR\right) e^{-rt} dt$$

$$s.t. \dot{S}_t = -R_t$$

$$\dot{Z}_t = aR_t - bZ_t$$

$$\overline{Z} - Z_t \geq 0$$

$$R_t \geq 0$$

$$S_t \geq 0$$

The model is an optimal control problem with one control variable R and two state variables S and Z. Compared with the model in Chakravorty, Magné and Moreaux (2006), this model can be seen as a very specific case where both clean backstop energy and abatement activities are not considered.

The constraint $R_t \geq 0$ requires that the extraction amount keeps positive during all periods. This condition is automatically satisfied due to the isoelasticity characteristic of the demand function. The demand curve goes infinitely to X-axis and Y-axis but never reaches. Thus R, no matter how close to zero, will never really reach zero. Therefore the resource will be exhausted in infinite time. Another constraint $S_t \geq 0$ indicates the current stock of the fossil fuel must not go negative. This is also satisfied by the equation that the total extraction in all period is equal to the initial stock, that is, $\int_0^{\infty} R_t d_t = S_0$.

Current value Hamiltonian is defined as:

$$H = R^{\alpha} - cR - \lambda R + \mu(aR - bZ)$$

The sufficient conditions for a maximum are that the Hamiltonian function be concave in the control variables and the state variables. Since here H is a combination of concave functions, it is concave in (S, Z, R).

The Lagrangian is given by: $L = R^{\alpha} - cR - \lambda R + \mu(aR - bZ) + q(\overline{Z} - Z)$

Necessary conditions for optimality are:

$$\frac{\partial L}{\partial R} = \alpha R^{\alpha-1} - c - \lambda + a\mu = 0$$

$$\dot{\lambda} = r\lambda - \frac{\partial L}{\partial S} = r\lambda \Leftrightarrow \lambda_t = \lambda_0 e^{rt}$$

$$\dot{\mu} = r\mu - \frac{\partial L}{\partial Z} = r\mu + b\mu + q = \left(r + b\right)\mu + q$$

$$q \geq 0 (q = 0 if \overline{Z} - Z > 0)$$

The necessary transversality conditions at infinity are presented a bit differently in some similar studies. The study in Withagen (1994), for example, deals with one control variable and two state variables problem when one of the state variables, the pollution stock, is also involved in a damage function that is included in the objective function. He described the necessary condition for optimality on the costate variable μ for pollutant stock as $\lim_{t\to\infty} e^{-\rho t}\mu(t) = 0$ (ρ has the same meaning as r in the models discussed here). Hoel and Kverndokk (1996) when studying the greenhouse externality problem added a necessary condition $\lim_{t\to\infty} e^{-rt}\lambda(t) = 0$ on the costate variable λ for the resource stock. The situation studied here is different from the above studies in that only the control variable appears in the objective function that is to be maximized.

Borrowing from Kamien and Shwartz (1981), the complete conditions at infinity for a current value infinite horizon control problem, using notations in the models above, are as follows:[5]

$$\lim_{t\to\infty} e^{-rt}\lambda(t) \geq 0, \quad \lim_{t\to\infty} e^{-rt}\mu(t) \geq 0$$

$$\lim_{t\to\infty} e^{-rt}\lambda(t)S_t = 0, \quad \lim_{t\to\infty} e^{-rt}\mu(t)Z_t = 0$$

[5]These conditions are also referred to as transversality conditions at infinity.

Chakravorty, Magné and Moreaux (2006) useed the latter two conditions. $\lim_{t\to\infty} e^{-rt}\lambda(t) = \lim \lambda_0 > 0$ holds since $\lambda_0 > 0$, the costate variable for resource stock must be positive. The stock of the resource goes to zero at infinity, $\lim_{t\to\infty} S_t = 0$, it thus can be shown that $\lim_{t\to\infty} e^{-rt}\lambda(t)S_t = \lim_{t\to\infty}\lambda_0 S_t = \lambda_0 \lim_{t\to\infty} S_t = 0$. Chakravorty, Magné and Moreaux have shown that μ_t after the binding time is actually zero. This is because after the binding period, the ceiling will not play any role in controlling the extracting process, thus the problem goes back to that without ceiling constraint. μ_t declines to zero when the binding period ends. Therefore, since $\mu_{(\infty)} = 0$, $\lim_{t\to\infty} e^{-rt}\mu(t) = 0$ and $\lim_{t\to\infty} e^{-rt}\mu(t)Z_t = 0$ are also satisfied.

In the path under ceiling, at an endogenous time point θ_1 the pollution stock is binding at the ceiling that is set by the policy. The ceiling keeps binding until a time θ_2. $q > 0$ is only satisfied during this binding period (θ_1, θ_2). $q = 0$ for $t < \theta_1$ and $t > \theta_2$. At time θ_1, the amount of carbon stock in the atmosphere reaches the regulated maximal $\overline{Z}$, its growth rate $\dot{Z} = aR - b\overline{Z} \equiv 0$ until the binding ends. Meanwhile $R \equiv \frac{b}{a}\overline{Z}$, the extraction amount keeps at this same level $\frac{b}{a}\overline{Z}$ from the beginning to the end of the binding period. During the other two periods when the ceiling is not binding, $(0, \theta_1)$ and (θ_2, ∞), $\mu_t = \mu_0 e^{(r+b)t}$ when $0 < t < \theta_1$; $\mu_t = 0$ for $t > \theta_2$. In addition, $\mu < 0$ for $t < \theta_2$. The absolute value of μ must be increasing and then decreasing to zero at time θ_2.

4.3 When extraction cost is zero

Consider a simple situation where $c = 0$. Stiglitz (1976) showed that under stationary, constant elasticity demand and zero extraction cost, the extraction path in the monopoly case is identical to that in the competitive case. This is because under the constant elasticity demand and zero cost, the marginal profit of the monopoly is proportional to price, thus the optimal marginal rate and price both grow at the rate of r, which is exactly the same as in competitive case.

The social utility function is defined by $U(R)=\int_0^R P d\tau = \frac{1}{\alpha} R^{\alpha}$. Compared with he profit function for monopoly, $Y(R)=R^{\alpha}$, it is easily seen that $Y(R)=\alpha U(R)$. Now with same exhaustibility and ceiling constraints, the social planner maximizes $\int_0^{\infty}\big(U(R)\big)e^{-rt}dt$, while the monopolist maximizes: $\int_0^{\infty}\big(Y(R)\big)e^{-rt}dt = \int_0^{\infty}\big(\alpha U(R)\big)e^{-rt}dt = \alpha\int_0^{\infty}\big(U(R)\big)e^{-rt}dt$.

The first order conditions for Lagrangian in both monopoly and competitive cases are: $\frac{\partial L}{\partial R}=\alpha p-\lambda+\mu=0$, $\alpha p_t=\lambda_t-\mu_t$, in addition $\dot{\lambda}=r\lambda$, $\dot{\mu}=r\mu+b\mu$ when the ceiling is not binding. Take derivative one can get $\alpha\dot{p}=(\dot{\lambda}-\dot{\mu})=(r\lambda-r\mu-b\mu)$, so $\frac{\dot{p}}{p}=r-\frac{b\mu}{\lambda-\mu}$. $\lambda>0$, and $\mu<0$ as pollution is a "bad", so $\frac{b\mu}{\lambda-\mu}<0$, thus $\frac{\dot{p}}{p}>r$.

It can be shown that $\frac{\dot{U}(R)}{U(R)}=\frac{\dot{Y}(R)}{Y(R)}=\frac{\dot{P}}{P}=r-\frac{b\mu}{\lambda-\mu}>r$. For both the monopoly and the competitive case, the price paths are identical. The reason for this is that the same optimal path would maximize a function as well as the same function that is multiplied by a parameter, thus in the existence of ceiling constraint, the monopoly and competitive behavior are also identical under constant elasticity demand and zero extraction cost assumption. As is shown, with a ceiling constraint, the price grows at a rate of $r-\frac{b\mu}{\lambda-\mu}$, which is higher than r, for both monopoly and competitive case. In the existence of the "maximal stock pollution policy", resource is depleted more rapidly than otherwise in the beginning period until the ceiling is binding. When the binding period stops, μ equals zero, the price will grow at a rate of r as it is in the case without ceiling constraint.

The path for the pollution stock under ceiling constraint would differ from the "natural growth" when no restriction is added on pollution stock. Since $\dot{Z}_{t=0} = aR_0 - bZ_0$, the comparison of the initial growth rate for pollution stock is the same as the comparison of the initial extraction amount. The higher R_0 is, the faster the stock of pollution grows initially. Consider two extreme cases: if $\overline{Z} = Z_0$, the ceiling is binding from the beginning and stock of pollution declines gradually after binding ceases; if $\overline{Z}$ is equal to the natural peak of the stock pollution, the two paths of Z with or without ceiling constraint are identical. The idea can be shown in figure 1.

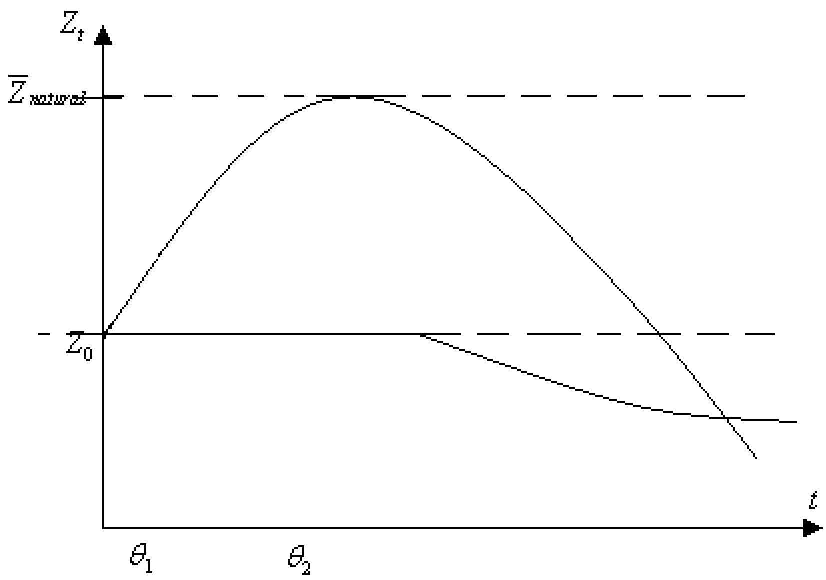

Figure 1 Paths for the pollution stock when $\overline{Z} = Z_0$ and when $\overline{Z} = \overline{Z}_{natural}$

Denote $\overline{R_0^{M.C}}$ as the initial extraction amount for the monopoly case, or equally, the competitive case, without ceiling constraint. As time 0 is assumed to be the time when the policy is announced, the path before time 0 may already lead to an extraction amount when reaching time 0. Suppose here at time 0 the plan is reconsidered with or without ceiling constraint, regardless of the previous path. In the case of no ceiling constraint, the reconsidered path will still be smoothly consistent with the previous path. The reason for this is that if a whole path is optimal for the entire horizon, then a section of the path still optimizes the objective function in the corresponding horizon section.

Denote $R_0^{M.C}$ as the initial extraction amount for both monopoly and competitive cases under ceiling constraint. Since the path is exactly the same for monopoly and for competitive market, take the monopoly case for instance. Without ceiling constraint, first order condition: $\alpha R^{\alpha-1} - \lambda = 0$, together with $\lambda = \lambda_0 e^{-rt}$ one can get $R = (\frac{\lambda_0 e^{-rt}}{\alpha})^{\frac{1}{\alpha-1}}$. Thus $\overline{R_0^{M.C}} = (\frac{\lambda_0}{\alpha})^{\frac{1}{\alpha-1}}$.

Using the resource scarcity: $\int_0^\infty R_t d_t = S_0 \Leftrightarrow \int_0^\infty (\frac{\lambda_0 e^{rt}}{\alpha})^{\frac{1}{\alpha-1}} dt = S_0 \Leftrightarrow (\frac{\lambda_0}{\alpha})^{\frac{1}{\alpha-1}} (\frac{\alpha-1}{r}) \left. e^{\frac{rt}{\alpha-1}} \right|_0^\infty = S_0$,

$(\alpha - 1 < 0) \Leftrightarrow \overline{R_0^{M.C}} = \frac{r}{1-\alpha} S_0$

With a ceiling constraint, first order condition is given by:

$$\alpha R_t^{\alpha-1} - \lambda_t + a\mu_t = 0 \Leftrightarrow R_t = (\frac{\lambda_t - a\mu_t}{\alpha})^{\frac{1}{\alpha-1}}.$$

For $0 < t < \theta_1$, $\mu_t = \mu^{(1)} = \mu_0 e^{(r+b)t}$, $R_t = R^{(1)} = (\frac{\lambda - a\mu^1}{\alpha})^{\frac{1}{\alpha-1}} = (\frac{\lambda_0 e^{rt} - a\mu_0 e^{(r+b)t}}{\alpha})^{\frac{1}{\alpha-1}}$;

For $\theta_1 < t < \theta_2$, $\dot{\mu}_t = (r+b)\mu_t + q_t \Leftrightarrow \mu_t = \mu^{(2)} = \mu_0 e^{(r+b)t} + e^{(r+b)t} \int e^{-(r+b)t} q(t) dt$,

$$R \equiv \frac{b}{a}\overline{Z};$$

For $t > \theta_2$, $\mu_t = 0$, $R_t = R^{(3)} = (\frac{\lambda}{\alpha})^{\frac{1}{\alpha-1}} = (\frac{\lambda_0 e^{rt}}{\alpha})^{\frac{1}{\alpha-1}}$

Through the horizon, $\lambda_t = \lambda_0 e^{rt}$

The initial extraction amount can be written as: $R_0^{M.C} = (\frac{\lambda_0 - a\mu_0}{\alpha})^{\frac{1}{\alpha-1}}$

In the binding period $\theta_1 < t < \theta_2$, the extraction amount keeps at the same level $\frac{b}{a}\overline{Z}$. This indicates that the rate of growth of R during the time must be zero, that is:

$$\alpha R^{\alpha-1} - \lambda + a\mu = 0 \Leftrightarrow \alpha(\alpha-1)R^{\alpha-2}\dot{R} = \dot{\lambda} - a\dot{\mu} = 0 \Leftrightarrow r\lambda_t - a(r+b)\mu_t - aq_t = 0$$

From this together with $\alpha(\frac{b}{a}\overline{Z})^{\alpha-1} = \lambda - a\mu$, one can get the expression of q as a function of λ_0 as:

$$q_t = \frac{r+b}{a}\alpha(\frac{b}{a}\overline{Z})^{\alpha-1} - \frac{b}{a}\lambda_t = \frac{r+b}{a}\alpha(\frac{b}{a}\overline{Z})^{\alpha-1} - \frac{b}{a}\lambda_0 e^{rt}$$

The dynamics of the stock of pollution: $\dot{Z} = aR - b\overline{Z} \Rightarrow Z_t = Z_0 e^{-bt} + e^{-bt}\int e^{bt}R(t)dt$.

There are four unknown parameters, $\theta_1, \theta_2, \lambda_0, \mu_0$ which can be determined by the following four equations:

$$\int_0^{\theta_1} R^1 dt + \frac{b}{a}\overline{Z}(\theta_2 - \theta_1) + \int_{\theta_2}^{\infty} R^3 dt = S_0$$

Left continuity of R_t and Z_t :

$$R^1_{(t=\theta_1)} = \frac{b}{a}\overline{Z}$$

$$R^3_{(t=\theta_2)} = \frac{b}{a}\overline{Z}$$

$$Z_{(t=\theta_1)} = \overline{Z}$$

The initial extraction $R_0^{M.C} = R_0^{M.C}(S_0, Z_0, \overline{Z})$

Compared with $\overline{R_0^{M.C}} = \frac{r}{1-\alpha}S_0$, two situations may happen:

If $R_0^{M.C} < \overline{R_0^{M.C}}$, the stock of pollution grows slower in the beginning than without ceiling constraint, and after binding period declines gradually (shown in figure 2a); Denote the initial price of $\overline{P_0}$ when the resource is extracted without ceiling restraint. The initial price is higher than $\overline{P_0}$ (shown in figure 3a).

If $R_0^{M.C} > \overline{R_0^{M.C}}$, the stock of pollution grows faster initially, and in a shorter time reaches the ceiling, after binding time gradually declines (shown in figure 2b), and the starting price is lower than $\overline{P_0}$ (shown in figure 3b).

As has been shown before, with ceiling constraint, the price before binding time will grow at a rate higher than otherwise at the rate r , and after the binding time goes back to traditional Hotelling case where price grows at the rate of r .

Di Maria and Werf (2008) have proved in a situation with one non-renewable resource and backstop available, “an announced emissions constraint cannot lead to an increase in emissions in the period between announcement and implementation”.[6] However, constraint on emissions and constraint on accumulated pollution stock are two different kinds of policies. Whether the ceiling constraint will lead to an initially higher or lower growth rate of the pollution stock than without ceiling constraint, depends on the ultimate value of $R_0^{M.C}(S_0, Z_0, \overline{Z})$ as discussed above, which is not explicitly solved here.

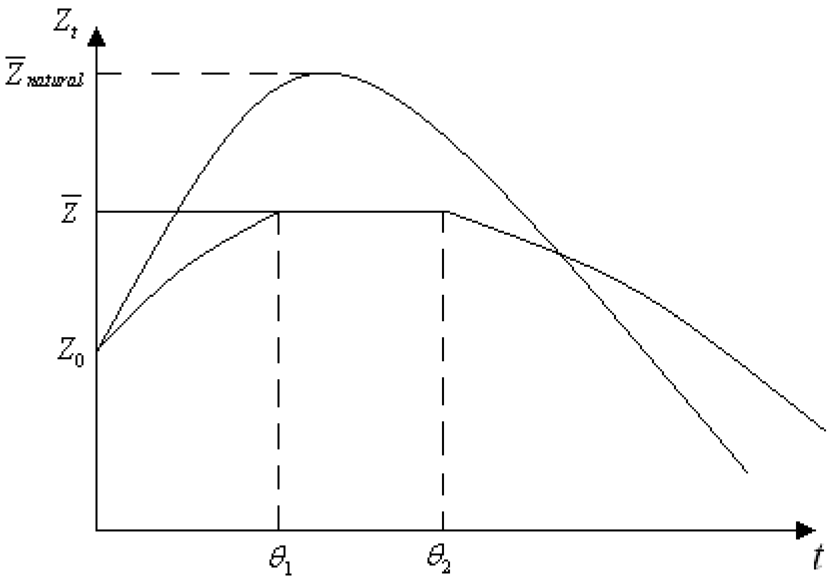

figure 2a Paths for the stock pollution with and without ceiling constraint when $R_0^{M.C} < \overline{R_0^{M.C}}$.

[6]See Di Maria and Werf (2008) section 6, Proposition 2 and Appendix B.2 for proof.

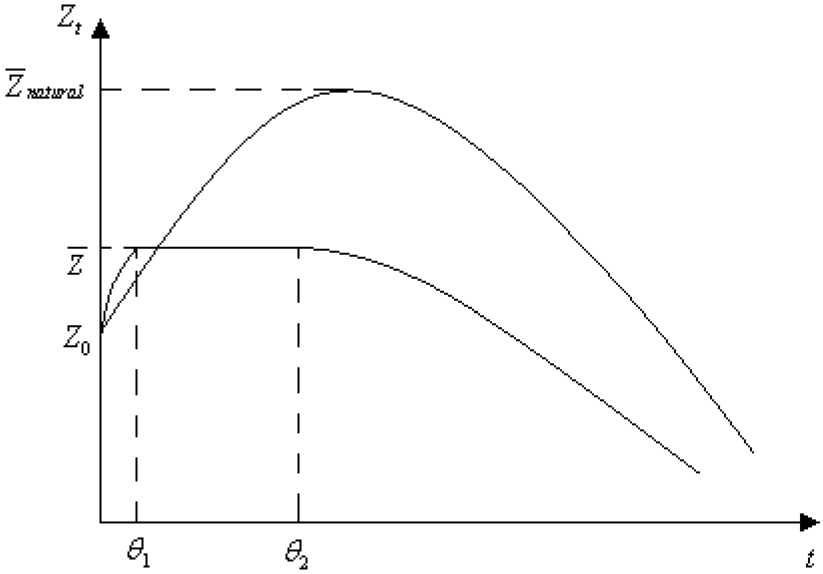

figure 2b Paths for the stock pollution with and without ceiling constraint when $R_0^{M.C} > \overline{R_0^{M.C}}$

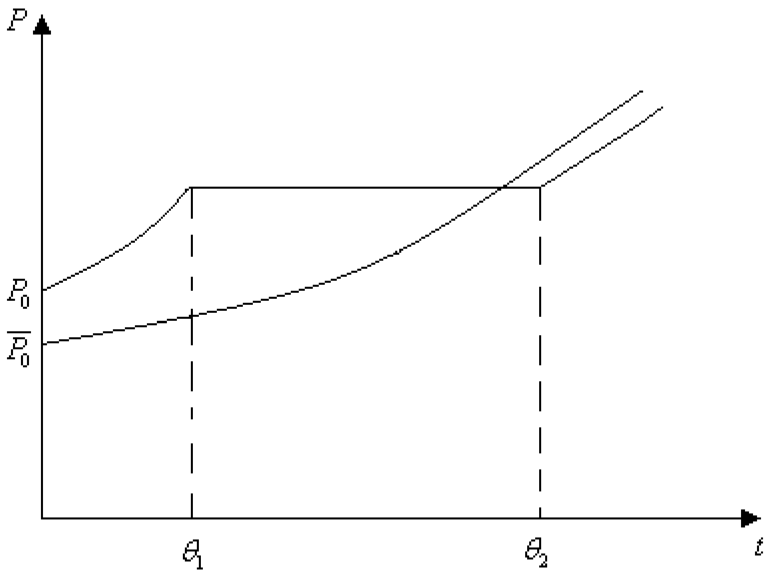

figure 3a The price paths with and without ceiling constraint when $R_0^{M.C} < \overline{R_0^{M.C}}$

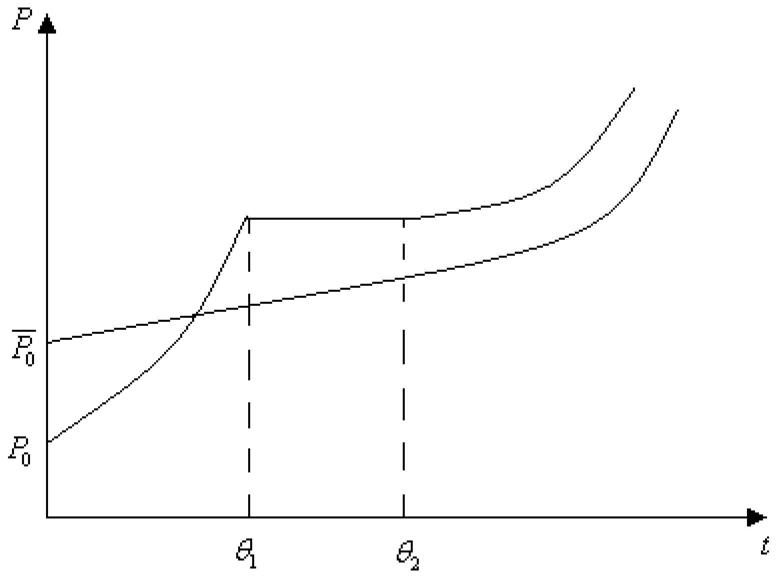

figure 3b The price paths with and without ceiling constraint when $R_0^{M.C} > \overline{R_0^{M.C}}$

4.4 The announcement and implementation of climate policy

When studying the effect of climate policy and the corresponding extraction path that is adjusted due to the regulation, the time of the announcement and the implementation of the policy are important to know in first place. Quantitatively, the accumulated amount of greenhouse gas in the atmosphere should not exceed the ceiling $\overline{Z}$ from the time of implementation. The policy maker can when set the ceiling $\overline{Z}$, requiring the pollution stock to "never exceed $\overline{Z}$ " or to "not exceed $\overline{Z}$ from a time point, say, year 2010". And in the latter case, it might make a difference that this time point is year 2010 or year 2100. Basically, the longer the period is from the announcement to the implementation, the more time is given to firms to prepare and adjust the production schedule.

The announcement effort is a new topic in the literature. So far only a few papers have investigated the effects of announced climate policy. Kennedy (2002) argues that the policies on emission reductions may be costly and inefficient during the period between announcement and implementation. Smulders and Werf (2007) show in the case of two resources, high and low in content of carbon, that the announcement of an emission constraint at a future date immediately causes a drop in the extraction rate of high carbon resource and a rush on resources that will be used less after implementation, which is contrast to the case without announcement when the both emissions and output jump the instant the constraint is introduced. Di Maria and van der Werf (2008) initiate a study on the effect of announcement on emissions of carbon dioxide, when an economy is facing a constraint such as the Kyoto Protocol. They have shown that when two perfectly substitutable fossil fuels are available, one high in carbon and the other low-carbon, announcement of the climate policy might have two opposite effects: it might reduce a policy burden since the pre-announcement has given firms time to adjust. Yet the announcement might cause an immediate increase in carbon dioxide emissions. So far the announcement effort of pollution stock constraint hasn't been specifically studied.

Under the same assumptions as the model discussed previously, suppose when announcing the ceiling constraint the policy maker also set a time for enforcement: the pollution stock must not exceed $\overline{Z}$ from, say, time $\overline{\theta}$. Assume the time at which the pollution stock equals an amount $\overline{Z}$ when there is no ceiling constraint is $\hat{\theta}_1$ and $\hat{\theta}_2$.

If $\overline{\theta}$ is set at time $\hat{\theta}_2$ or after, then it does not affect the original path at all and there would be no binding time, the path for stock pollution will be exactly the same as when there is no ceiling constraint.

If $\overline{\theta}$ is equal to 0, the implementation comes into force the moment it is announced, and the result will be the same as in the previously discussed model, where price grows at a faster rate than otherwise r in the Hotelling path, because if it is not so, as has been explained previously, there would be a sudden jump or drop down in the extraction amount, such kink in the extraction path will contradict the continuity requirement for maximum. At time θ_1 the pollution stock reaches the ceiling and keeps binding until θ_2, then gradually declines afterwards.

If $\overline{\theta}$ is set between 0 and $\hat{\theta}_2$, it might happen that the resource owner produces from very high amount initially and extracts the resource even faster, because during the time period $(0,\overline{\theta})$ he is not restricted by the ceiling constraint. Whether doing so will actually maximize his profit is not studied here. The idea is shown in figure 4. The solid line represents the path without ceiling constraint and with ceiling constraint when policy maker sets the implementation time $\overline{\theta} \geq \hat{\theta}_2$. The dashed line represents the optimal path under ceiling constraint for $\overline{\theta} = 0$. The dotted line represents the possible path when $0 < \overline{\theta} < \hat{\theta}_2$.

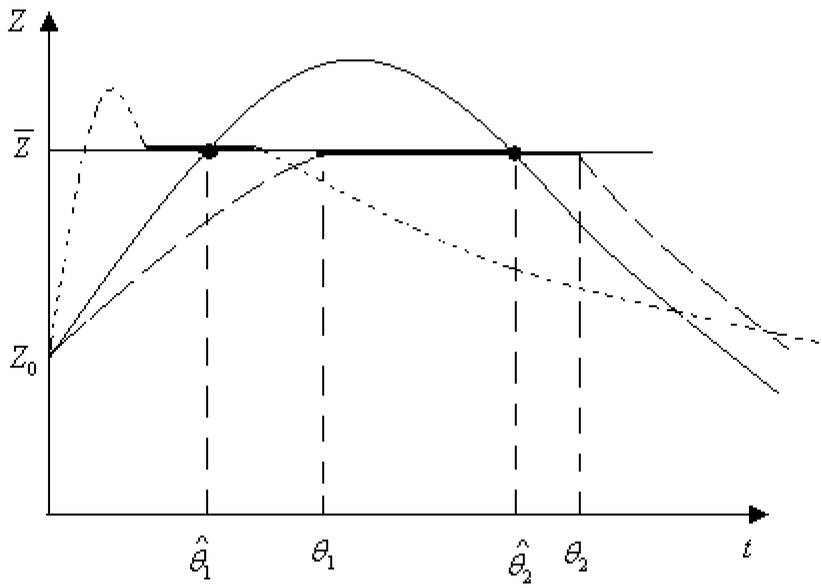

figure 4 Possible paths for the pollution stock with different implementation time

Notice that the climate policy here allows an initial increase in carbon stock and require it to not go beyond a level that is higher than the initial stock pollution in future, that is, $\overline{Z} \geq Z_0$. This is by nature different from when the policy is aimed to stabilize the carbon stock at a lower level than the initial stock pollution from at a future time, which in reality is the target of Kyoto Protocol.

A very particular case would be the ceiling is binding from the very beginning $t = 0$ when it is announced. Smulders and Werf (2007) have shown that if this is true in a situation with both high carbon and low carbon resources, then the constraint will not bind forever since stock resources, from which emissions stern, are depleted over time. Here we study a single resource without substitute resource. If the ceiling is binding initially, $\theta_1 = 0$, $R_0^{M.C} = \frac{b}{a}\overline{Z}$, and the optimality conditions actually becomes two unknown parameter λ_0, θ_2 satisfying the following equations:

$$(\frac{b}{a}\overline{Z})\theta_2 + \int_{\theta_2}^{\infty} R_t dt = S_0$$

$$R_{t=\theta_2} = (\frac{\lambda_0 e^{r\theta_2}}{\alpha})^{\frac{1}{\alpha-1}} = \frac{b}{a}\overline{Z}$$

The optimal time for stopping binding the ceiling is solved as:

$$\theta_2^* = \frac{aS_0}{b\overline{Z}} + \frac{\alpha - 1}{r}$$

The result has a similar form of that solved by Smulders and Werf, except that in latter case there are two initial stocks of fossil fuels. The optimal stop time for binding depends negatively with the ceiling $\overline{Z}$. The higher the ceiling is, the short time it takes to stop binding.

Generally, under constant elasticity of demand, when the extraction cost is zero, the monopoly extraction path is identical with that of the perfect competitive case. The existence of the pollution stock ceiling will result in both monopoly and competitive case that price grows at a higher rate than the discount rate otherwise. The initial growth rate of the pollution stock could be either higher and lower than the natural pollution stock growth, due to the fact that the initial extraction amount compared with no ceiling constraint situation, is to be determined by the initial pollution and the maximal ceiling in addition to the stock of the resource. This is also the reason that the initial price could also be both higher and lower than without ceiling constraint.

4.5 When there is constant positive marginal extraction cost

Comparing with the monopolist, the social planner is aiming to maximize a total net social utility, which equals the consumer surplus minus the extraction cost. Control problem for competitive case:

$$\max \int_0^\infty \left(\frac{1}{\alpha} R^\alpha - cR \right) e^{-rt} dt$$

$$\dot{S}_t = -R_t$$

$$\dot{Z}_t = aR_t - bZ_t$$

$$\overline{Z} - Z_t \geq 0$$

$$R_t \geq 0$$

$$S_t \geq 0$$

Recall that Hotelling (1931) demonstrated that with a constant marginal extraction cost, the optimal path would be such that the price minus marginal cost rises at the rate of discount in the competitive market, and marginal revenue minus marginal cost (resource rent) rises at the rate of discount in a market of monopoly.

When extraction cost is taken into consideration, monopoly and competitive paths would differ from each other. Stiglitz (1976) showed that with extraction cost, the monopolist would bias to behave more conventionally, that is, the extraction would grow at a lower rate than the social optimal in the beginning. Following this result, the paths for monopoly and competitive extraction and the relative pollution stock change are shown in figure 5a and 5b. Without ceiling constraint, the monopolist will take the conventional policy, the extraction rate will be lower than in the competitive case. Since the initial extraction amount of monopoly is lower, the stock of pollution in monopoly case will grow in a lower rate initially than that of competitive case. The same level of ceiling that functions in competitive case may not make effect in monopoly case. So with a ceiling constraint, it might be binding in the competitive case but not in the monopoly case if the ceiling is, although lower than the natural cap of competitive case, still above the monopoly natural cap.

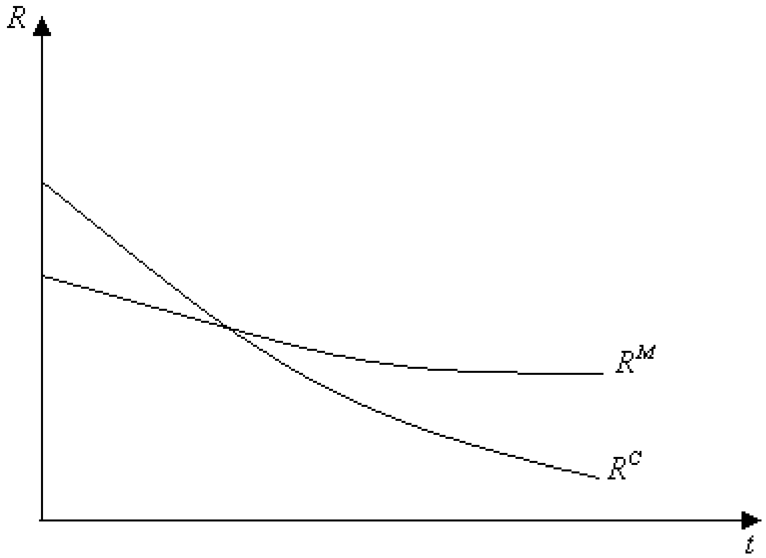

figure 5a   Comparison of extraction paths for monopoly and competitive market

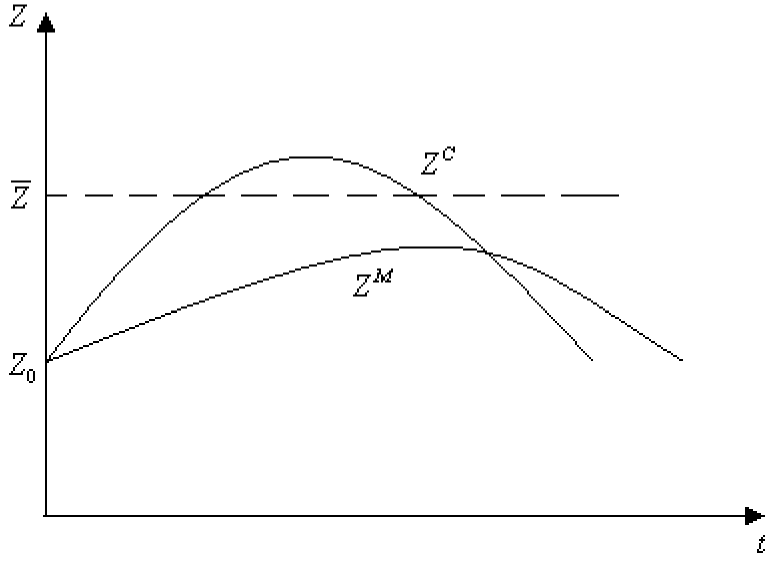

figure 5b   Comparison of paths for stock pollution in monopoly and competitive market

4.6 Comparison monopoly versus competitive market

Notice that the constraints are same for both the monopoly and the competitive. The objective function for monopoly:

$$R^\alpha - cR = \alpha.\left(\frac{1}{\alpha}R^\alpha - (\frac{c}{\alpha})R\right).$$

While the social objective function is $\frac{1}{\alpha}R^\alpha - cR$, where c is the positive constant marginal cost.

The monopoly and the competitive paths would be again the same if the cost for monopoly is α time the cost in competitive case, that is,

$$R^\alpha - c^M R = \alpha.\left(\frac{1}{\alpha}R^\alpha - (\frac{c^M}{\alpha})R\right) = \alpha.\left(\frac{1}{\alpha}R^\alpha - (\frac{\alpha c^C}{\alpha})R\right) = \alpha.\left(\frac{1}{\alpha}R^\alpha - c^C R\right)$$

But in reality, the cost for the social and monopoly would be same under the same technological conditions. Suppose the costs are same for social and monopoly case. The objective functions for monopoly and social optimum are: $\alpha.\left(\frac{1}{\alpha}R^{\alpha}-(\frac{c}{\alpha})R\right)$ and $\left(\frac{1}{\alpha}R^{\alpha}-cR\right)$ respectively. Since $0<\alpha<1$, the comparison of monopoly and social optimum is identical to the comparison of "social optimum with high cost" and "social optimum with low cost". That is, moving to monopoly from a social optimum is equal to moving to social optimum with low cost from social optimum with high cost

This cost can be the extraction cost. It can also be seen as the constant unit abatement cost. The carbon stock in the atmosphere can be reduced through costly abatement. Suppose there is a positive constant unit abatement cost, the higher this abatement cost is, the more costly it is to reduce the carbon emission through abatement activities.

Conclusions

This paper reviews some main studies on fossil fuel extraction under climate issues and studies a theoretical model of monopoly extraction under ceiling constraint. So far almost none research has been done dealing with the effect of a climate policy on a resource monopoly. Our results show that under constant elasticity demand and zero extraction cost, the monopolist will behave exactly the same as in the competitive case, and the existence of the ceiling constraint will initially push the extraction to grow at a rate higher than the interest rate in both monopoly and competitive case; With a non-zero extraction cost the monopoly may be under lower risk to be affected by the ceiling than the competitive case; With a constant non-zero cost, the comparison monopoly versus competitive market is identical to the comparison “social optimum with high cost” versus “social optimum with low cost”. Furthermore, this cost can be both extraction cost and abatement cost.

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